Learning stability guarantees for data-driven constrained switching linear systems

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Introduction

We focus on stability analysis of *constrained switching linear systems* in which the dynamics are unknown and whose switching signal is constrained by an automaton. We propose a *data-driven* Lyapunov framework for providing probabilistic stability guarantees based on data harvested from observations of the system. Moreover, we show that the *entropy* of the language accepted by the automaton allows to bound the number of samples needed in order to reach some pre-specified accuracy.

1 Constrained switching linear systems

We consider discrete-time *switching linear systems* defined by a set $\mathscr{A} = \{A_i\}_{i\in\{1,\ldots,m\}}$ of m matrices. Their dynamics, for any $t \in \mathbb{N}$, is given by the following equation:

$$x_{t+1} = A_{\sigma(t)} x_t. \tag{1}$$

A *constrained switching linear system* (*CSLS*) is a switching linear system with logical rules on its switching signal. We represent these rules by an *automaton* i.e., a strongly connected, directed and labelled graph $\mathscr{G}(V,E)$ with V the set of nodes and E the set of edges. The edge $(v,w,\sigma) \in E$ between two nodes $v,w \in V$ carries the *label* $\sigma \in \{1,\ldots,m\}$, which maps to a mode of the switching system.

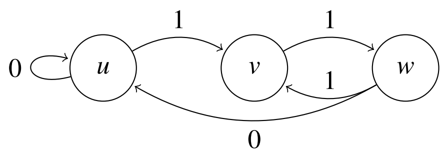

Figure 1: An automaton defined by two modes (0 and 1), and the following logical rule. If mode 1 is chosen, then it has to be chosen at least twice in a row.

The *constrained joint spectral radius* $\rho(\mathscr{G},\mathscr{A})$ of a given CSLS is defined as follows:

$$\begin{aligned}\rho(\mathscr{G},\mathscr{A}) = \lim_{t\to\infty} \max\{\|A_{\sigma(t-1)}\ldots A_{\sigma(0)}\|^{1/t} :\\ \sigma(0),\ldots,\sigma(t-1) \text{ is accepted by } \mathscr{G}\}.\end{aligned} \tag{2}$$

The stability of a CSLS defined by $\mathscr{A}$ and $\mathscr{G}$ is ruled by its constrained joint spectral radius. It is asymptitocally stable if and only if $\rho(\mathscr{G},\mathscr{A}) < 1$ (see [2], Corollary 2.8).

Finally, the *entropy* $h(\mathscr{G})$ of an automaton $\mathscr{G}$ is the growth rate of $|\mathscr{L}_{\mathscr{G},l}|$ where $\mathscr{L}_{\mathscr{G},l}$ is the language accepted by $\mathscr{G}$ restricted to length l (see [3], Definition 4.1.1). It is given by the equation

$$h(\mathscr{G}) = \lim_{l\to\infty} \frac{\log_2 |\mathscr{L}_{\mathscr{G},l}|}{l}. \tag{3}$$

2 Data-driven approach

In many practical applications, the engineer cannot rely on having a model, but rather has to analyse stability in a *data-driven* fashion. In this setting, we have acces to a set of observations $\omega_N = \{(x_{i,0}, A_i), i = 1,\ldots,N\}$, where $x_{i,0} \in \mathbb{S}$, the unit sphere, and $A_i = A_{\sigma_i(l-1)} \ldots A_{\sigma_i(0)}$ with $\sigma_i(0),\ldots,\sigma_i(l-1)$ accepted by $\mathscr{G}$.

In order to tackle hybrid behaviors in arbitrary switching linear systems, novel data-driven stability analysis methods have been recently developed based on scenario optimization. For example, [1] provides a data-driven method for approximating the *joint spectral radius* of an unknown switching linear system. In this work, we generalize this method to CSLS. We derive a probabilistic Lyapunov method to approximate the *constrained joint spectral radius* in order to give probabilistic certificates on stability.

We then analyze further the obtained result. We show that a lower entropy allows for a better probablistic guarantee for the stability of a given CSLS.

References

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